Printed Pages : 4

AS-303

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 990303

Roll No.

B. Tech.

(SEM. III) (ODD SEM.) THEORY EXAMINATION, 2014-15

ENGINEERING MATHEMATICS – III

Time : Hours] [Total Marks : **100**

UNIT - 1

1 Answer any **four** from the followings : **(4×5=20)**

1 If $f(z)$ is a regular function of z, then prove that

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2.$$

2 Find the analytic function $f(z)=u+iv$, given that $v = e^x(x\sin y + y\cos y)$

3 Evaluate the following integral using Cauchy's integral formula $\int_C \frac{4-3z}{z(z-1)(z-2)}dz$ where C is the circle $|z|=\frac{3}{2}$.

4 Expand $f(z)=\frac{1}{(z-1)(z-2)}$ for $1<|z|<2$.

5 Determine the poles of the following function and residue at each pole :

$$f(z)=\frac{z^2}{(z-1)^2(z+2)}$$ and hence evaluate

$$\int_C \frac{z^2dz}{(z-1)^2(z+2)} \quad \text{where} \quad C:|z|=3$$

6 Evaluate $\int_0^{2\pi}\frac{d\theta}{2+\cos\theta}$ the by contour integration in the complex plane.

UNIT - 2

2 Answer any **four** from the followings : **(4×5=20)**

1 Find Fourier sine transform of $f(x)=\frac{e^{-ax}}{x}$.

2 Using Parseval's identity, show that $\int_0^{\infty}\frac{x^2dx}{(x^2+1)^2}=\frac{\pi}{4}$.

3 Solve the equation $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}, x>0, t>0$ subject to the condition

(i) $u=0$ when $x>0, t>0$

(ii) $u=\begin{cases}1, 0<x<1\\0, x\geq 1\end{cases}$ when $t=0$

(iii) $x(x,t)$ is bounded.

4 Solve the difference equation

$y_{k+1}-2y_{k-1}=0, k\geq 1, y_{(0)}=1$.

5 Find the Z-transform of $\sin\alpha k, k\geq 0$.

6 Find $Z^{-1}\frac{9z^3}{(3z-1)^2(z-2)}$.

3 Answer any **four** from the followings : **(4×5=20)**

1 Three urns contains 6 red, 4 black; 4 red, 6 black; 5 red, 5 black balls respectively. One of the urns is selected at random and a ball is drawn from it. If the ball drawn is red, find the probability that it is drawn from the first urn.

2 Using Poisson distribution, find the probability that the ace of spades will be drawn from a pack of well-shuffled cards at least once in 104 consecutive trials.

3 Find the mean and standard deviation of Normal distribution.

4 A manufacturer of envelopes knows that the weight of the envelopes is normally distributed with mean 1.9 gm and variance 0.01 gm. Find how many envelopes weighing (i) 2 gm or more, (ii) 2.1 gm or more, can be expected in a given packet of 1000 envelopes. [Given : if t is the normal variable, then $\phi(0\leq t\leq 1)=0.3413$ and $\phi(0\leq t\leq 2)=0.4772$]

5 Find the moment generating function of Binomial distribution about its mean.

6 If the probability density function of a random variable x is

$$f(x)=\begin{cases}kx^{\alpha-1}(1-x)^{\beta-1}, 0<x<1, \alpha>0, \beta>0\\0, otherwise\end{cases}$$

Find k and mean of x.

4 Answer any **two** from the followings : **(2×10=20)**

1 If an approximate root of the equation $x(1-\log_e x)=0.5$ lies between 0.1 and 0.2, find the value of the root correct to three decimal places by Newton-Raphson method.

2 Solve the system of equations

$x+y+54z=110$

$27x+6y-z=85$

$6x+15y+2z=72$

Using Gauss-Seidel iteration method.

3 Find the cubic spline approximation for the function $y=f(x)$ from the following data, given that $y_0'=y_3'=0$

x	-1	0	1	2
y	-1	1	3	35

5 Answer any **two** from the followings : **(2×10=20)**

1 The velocity V of a particle at distances from a point on its path is given by the table :

S	0	10	20	30	40	50	60	feet
V	47	58	64	65	61	52	38	Feet/sec

Estimate the time taken to travel 60 feet by using Simpson's one-third rule. Compare the result with Simpson's $\frac{3}{8}$ rule.

2 By applying the fourth order Runge-Kutta Method find $y(0.2)$ from $y'=y-x, y(0)=2$ taking $h=0.1$

3 The differential equation $\frac{dy}{dx}=y-x^2$ is satisfied by $y(0)=1, y(0.2)=1.12186,$ $y(0.4)=1.46820,$ $y(0.6)=1.7379.$ Compute the value of $y(0.8)$ by Milne's predictor-corrector formula.